PSEUDO-ALIAS DISTORTIONS IN SAMPLED DATA SYSTEMS

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ABSTRACT
Investigations of aliasing effects in digital waveform sampling will show the existence of a pseudo-alias domain lying to the left of a "Nyquist line" in a plane defining the boundary between two domains of sampling. To the right of the line lies the classic alias domain. We discuss a problem with some band-limited signals below the Nyquist limit, when displayed output can also show a false modulation envelope whenever the sample rate and the signal frequency are related by ratios of mutually prime integers. An improved definition of the Nyquist frequency is proposed.

I. Signal basis

The Nyquist Frequency[1][5] is the most fundamental of a large set of signal frequencies following the special rule in Equation 1 not found in industry literature. The Nyquist-Shannon Sampling Theorem states that a signal can be completely constructed if the sampling frequency is equal to or above twice the Nyquist Frequency[2]. However, it is common for commercial waveform display instrumentation to just display the real-time results of the sampled waveform without performing any time intensive reconstruction. The user is left to discern whether the displayed waveform is adequately represented by what is visually observed[3]. Some equipment will filter the stream of samples by filling the gaps between samples with simple analog filters in order to make the waveform display presentable. Examples of the latter are types of audio playback devices such as CD players and cell phones.

For a pure single frequency sinewave signal, visually apparent sampled waveform distortion can occur whenever the signal frequency and the sample rate are related by the following relation

$$f = (m/n)f_s \tag{1}$$

where real numbers m and n follow the rule that m/n is less than 1/2, f is the signal frequency and f_s is the sample rate. Figure 1 provides some examples.

The Nyquist Frequency is located where m==1 and n==2. The Nyquist frequency is commonly stated as[1]:

The Nyquist frequency (cycles per second) is the frequency whose cycle-length (or period) is twice the interval between samples, thus 0.5 cycle/sample.

The above definition is commonly stated in various versions in the literature. We propose below a more concise restatement of the definition for Nyquist frequency after some discussion.

To review, an electronic sine wave signal of constant frequency (cycles per second) and constant peak amplitude of unity is sampled using an Analog-to-Digital Converter (ADC) device. The digital sample values are stored in a table of measured values in computer memory or in a computer file. Think of the samples as the mathematical equivalent of pieces of continuous electronic waveforms physically available in a system such as an audio amplifier or radio receiver that is supplying varying voltages to the digital sampler for further processing. The sampling frequency will be assumed to be definable as any steady frequency which can be varied with an adjustable "knob". For the purpose of this paper a signal is assumed created and studied at will using a computer spreadsheet to inexpensively model the results of samples acquired with expensive lab equipment or hardware in the field.

Again, for a signal frequency of f, $2f$ is the Nyquist frequency per the above definition. But there is a problem with referencing exactly $2f$. Assume the case where the samples of the signal are taken exactly at the zero crossings of the signal (two samples for each cycle of the signal) and graphed. Every other sample is taken when the signal is positive going and the next sample is taken when the signal is negative going. Obviously, all the samples will be forever zero at those points. In this case, no information will be available about the shape of the waveform. Samples that are all zero could just be the case of sampling a null signal that is a flat line. Any difference between the two cases will be indistinguishable.

In another case, consider where the two samples are taken at a 45 degree point and a 225 degree point of the same signal and graphed. For that graph we might claim the waveform is a triangle wave of amplitude 70.7... percent (sine of 45 degrees) of the actual signal. Exactly how to use any samples taken at the Nyquist frequency is not at all clear.

II. First improvement

In order to gain access to the actual shape of the waveform something must be done about the sample rate. We can raise the sample rate by some very small amount and then after enough samples are taken, the true shape of the signal might become obvious when graphed. When taken to the extreme, for example, lowering the signal frequency by one billionth of the value of the defined Nyquist frequency could ultimately assist in defining the true signal waveform shape. But without careful interlacing of the samples the frequency of the signal will appear to be wrong. Figure 2 shows a computed representation of what the signal may appear to be near a zero crossing when the sampling frequency is slightly increased. A conclusion is that for any faster sampling frequency above the Nyquist frequency there can be a way to discover the true waveform shape given enough digital signal processing.

But because $2f$ is special, at the very least a modification of the definition of the Nyquist frequency is required, using the definition of supremum[4][5]. To wit:

The Nyquist frequency (cycles per second) is the frequency whose cycle-length (or period) is the supremum of all usable sample rates satisfying an interval of at least 0.5 cycle/sample.

III. Prior work

Many years ago when digital instrumentation was coming into vogue, during design of a new commercial digital strip chart recorder, an analog sine wave signal generator was randomly swept up and down in frequency as a signal for the recorder. On the chart paper unexpected deviations from constant amplitude sine wave were occasionally being plotted. During the test the signal generator frequency was kept well below the Nyquist frequency sample rate. After a bit of study we postulated that the envelope distortion occurred per Equation 1 in the case of m = 6 and n = 53. Notably, 6/53 is quite a bit less than 1/2. The test was repeated using a commercially available digital storage oscilloscope and the resulting plotted display matched the strip chart recorder results. Recently, with no access to any lab equipment, we plotted a spreadsheet simulation of the same test waveforms. The plot matches exactly the results displayed on the strip chart recorder and oscilloscope. This is shown in Figure 3.

Further experimental testing of m and n reveals that as long as

$$m < (1/2)\, n \qquad 2$$

with real numbers m and n, there are many combinations that result in significant waveform distortions. Multiple examples of that effect appear in Figure 1. There are many more combinations available beyond those shown.

IV. Representative distortions

The sampled waveform system constrained for review here follows the Nyquist–Shannon sampling theorem, where for the additional representation of the signal

$$f(t) = \sin(\omega t) \qquad 3$$

and for frequency f where

$$\omega = 2\pi f \qquad 4$$

f must be less than 1/2 f_s , the sample rate[3]. We studied cases where, for integers m and n, m and n are mutually exclusive products of prime integers. They must be mutually exclusive so that

$$f < (m/n) f_s \qquad 5$$

where

$$m < n/2 \qquad 6$$

In the aforementioned lab testing, with the example where m/n = 6/53, the sine wave envelope ripple error size can be denoted as ε as in Figure 3. For sine wave signals of unity peak, ε was derived.

$$\varepsilon = 1 - \cos(\pi m/n) \qquad 7$$

Unsurprisingly, experiments with real hardware, or these software simulations, show that sampling systems easily confirm the often-quoted rule that true-to-life waveform displays need to have m very much less than n. Between that regime and m slightly less than n/2, there are thousands of examples of highly distorted views of signal waveforms in the visible waveform presentations.

V. Summary

While the discussion here has covered only cases of single frequency signals having a peak amplitude of unity, most real-world laboratory and field work involves complex waveform signals. Complex signals may consist of many frequencies and amplitudes happening all at the same time. Real-world signals may contain frequencies that can be affected by the sensitivity to the m/n ratio and perceptible distortions may result. An example is the effect of Compact Disk sample rate on the sampling and replay of audio music signals[3].

CONCLUSIONS:

The reader is encouraged to use test instrumentation or a spreadsheet to confirm the above claims. Evaluate whether any of these effects might occur in his/her experimental or manufactured hardware using digital signal processing. A suggestion of a useful upper limit to m/n is the commonly stated nominal 1:10 ratio.

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[1] *John W. Leis (2011). Digital Signal Processing Using MATLAB for Students and Researchers. John Wiley & Sons. p. 82. ISBN 9781118033807.* The Nyquist rate is twice the bandwidth of the signal ... The Nyquist frequency or folding frequency is half the sampling rate and corresponds to the highest frequency which a sampled data system can reproduce without error.
[2] *Shannon, Claude E. (January 1949). "Communication in the presence of noise". Proceedings of the Institute of Radio Engineers.* ***37*** *(1): 10–21. doi:10.1109/jrproc.1949.232969. S2CID 52873253.* Reprint as classic paper in: *Proc. IEEE*, Vol. 86, No. 2, (Feb 1998) Archived 2010-02-08 at the Wayback Machine. p. 448
[3] https://ntrs.nasa.gov/citations/20020027355
or
https://ia800307.us.archive.org/20/items/nasa_techdoc_20020027355/20020027355.pdf
[4] *Rudin, Walter (1976). "Chapter 1 The Real and Complex Number Systems". Principles of Mathematical Analysis (print) (3rd ed.). McGraw-Hill. p. 4. ISBN 0-07-054235-X.*
[5] Nyquist Frequency. (14 June 2026). In *Wikipedia*.
https://en.wikipedia.org/wiki/Nyquist_frequency

NOTE: This paper completely composed without any usage of Artificial Intelligence.

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Revision History
Revision 1.0
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    Added note about Artificial Intelligence

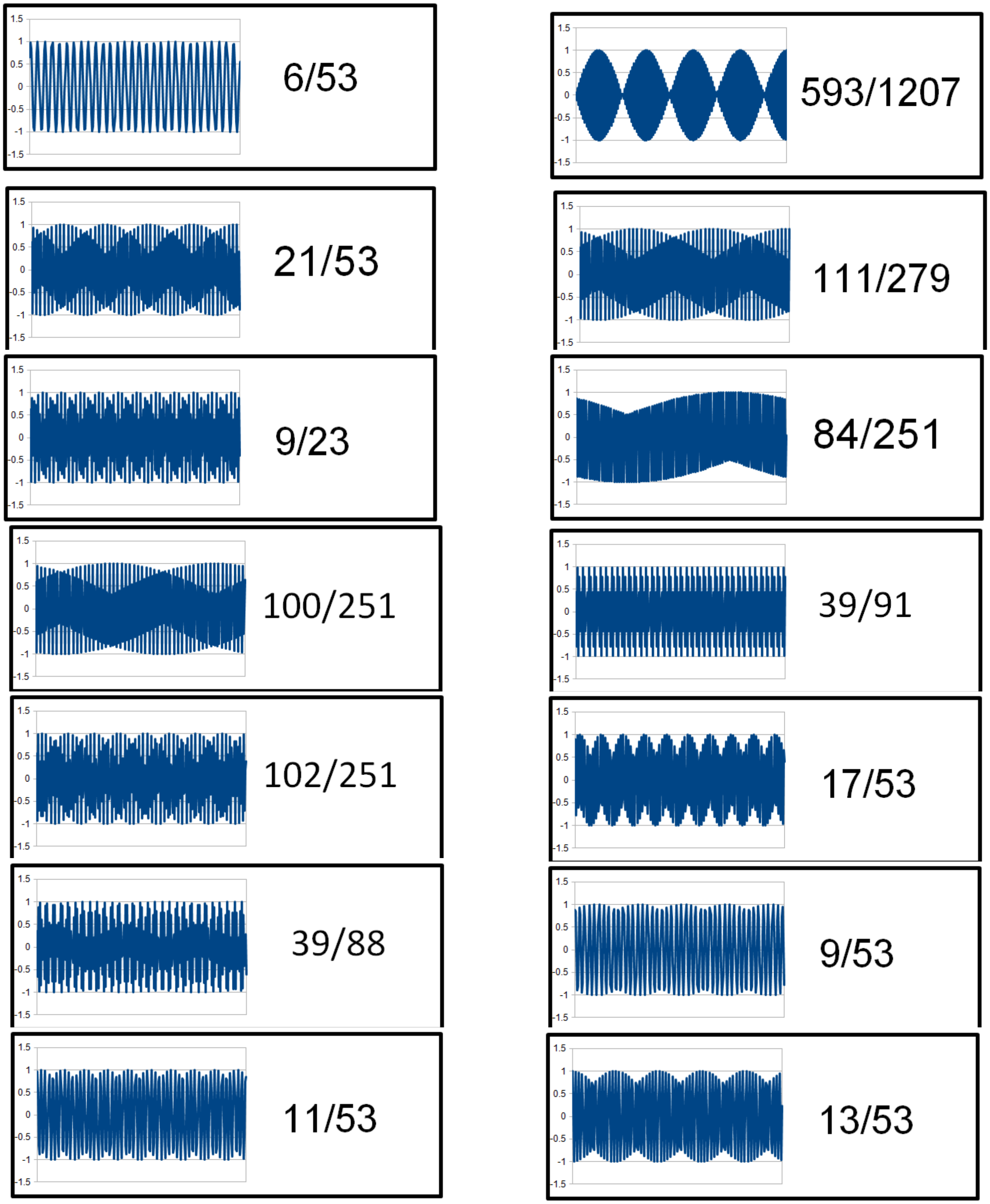

Figure 1 Showing representative samples with different ratios of integers m/n, where m is less than half of n. Appearances change upon expansion of time axis.

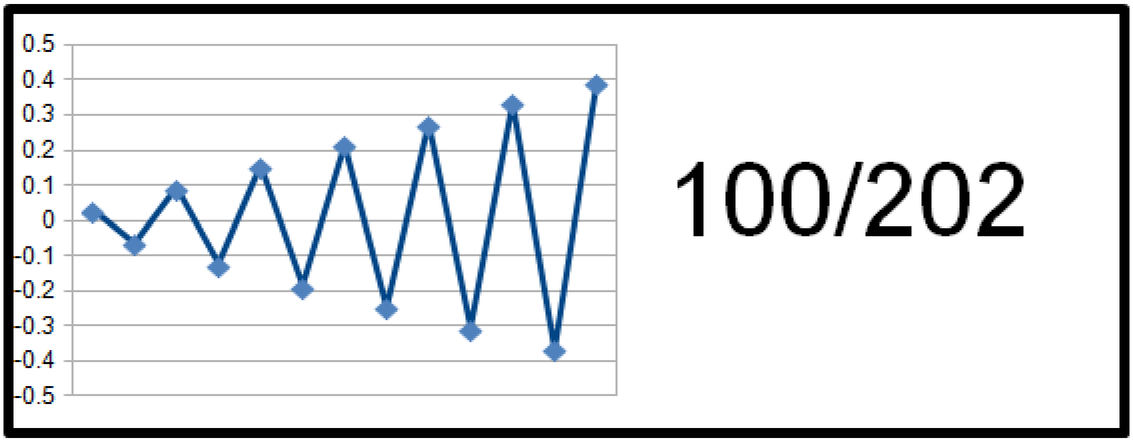

Figure 2

Spreadsheet simulation of strip chart or digital storage oscilloscope waveform sampled slightly below the Nyquist Frequency. Here the ratio m/n is 100/202. Appearances change upon expansion of time axis.

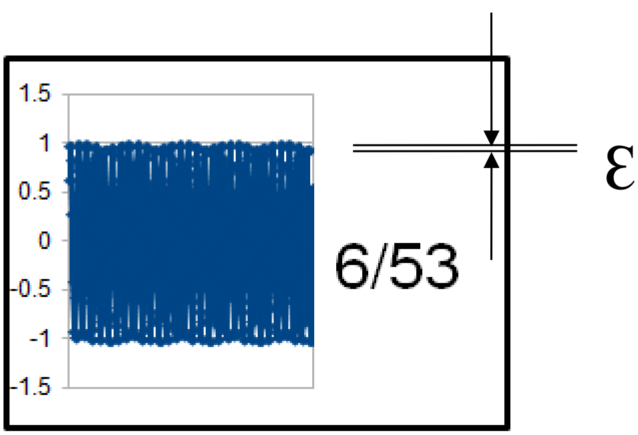

Figure 3

Spreadsheet simulation of actual strip chart or digital storage oscilloscope waveform samples used in reference to equation 7. Appearances change upon expansion of time axis.